Electrons Ejected from Aluminum by 150kVp-300kVp X-Rays

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150—300kVp X線に照射されたアルミニウムから放出される2次電子

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連続X線による2次電子放出と云う性質に関してアルミニウムが骨と等価であるか否かを確めるために，次の様な実験的研究を行つた．すなわち，150～300kVp X線を厚さ1㎜のアルミニウム板に垂直に入射させたとき，この板の背面から飛出す2次電子群のエネルギー分布および角度分布を電磁石型電子線分析器で測定した．この結果を以前に人骨で得た結果と比較したところ，2次電子のエネルギー分布に注目する限りではアルミニウムは骨に充分等価であるが，放出2次電子数については等価とは云えないことが判つた．

つぎに，放出2次電子の線束を2，3の近似計算法によつて算定し，実験結果と比較したところ，2次電子線束のX線線質による変化は唯一つの近似式だけで説明できないことが判つた．とくに，光電効果の寄与が大きい場合には2次電子放出の角度依存性を考慮した計算が必要である．

1. INTRODUCTION

The interaction of X-rays with matter emits secondary electrons which cause the successive physical, chemical or biological effects. Hence the basic physical information about the numbers and energies of the secondary electrons is needed for the interpretation of chemical and biological effects of X-rays or for purposes of dosimetry[3)4)].

Recently, the authors investigated experimentally the energy spectrum of electrons which emerged from the surface of a plane slab of bone or lucite while being irradiated by a continuous spectrum of X-rays, and then applied the results to the determination of the absorbed dose in a transition region near bone or lucite[7)]. Subsequently, emergent electrons from aluminum have been measured by the same method so as to examine whether aluminum could be equivalent to bone in respect of the nature such as the emission of secondary electrons. The results are described in the present paper. Moreover, some approximate calculations are also presented of the electron flux emerging from the aluminum slab exposed to continuous X-rays, being compared with experimental results.

2. EXPERIMENTAL METHOD

The principle of the experimental procedures was the same as that used previously to study the energy spectrum of electrons ejected from bone or lucite by X-rays[7)]. The present work was carried out

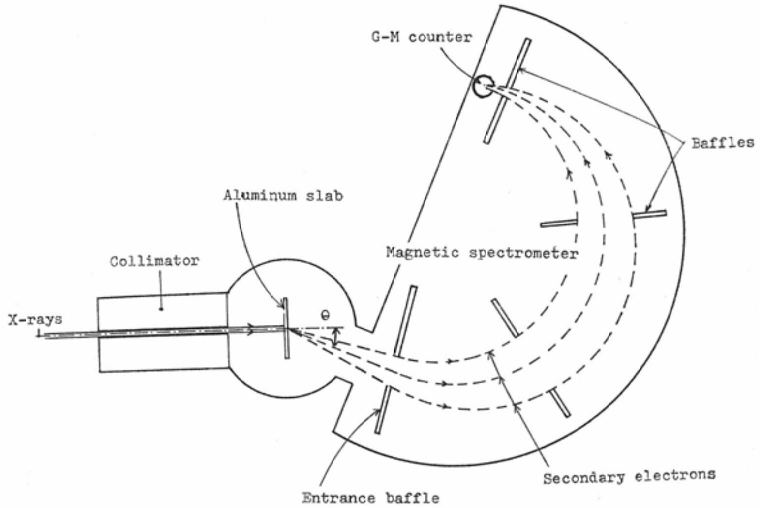

Fig. 1. Schematic diagram of the experimental arrangement.

for 1 mm thick aluminum slab. The essentials of the experimental septup are sketched in Fig. 1. When the aluminum slab was normally exposed to a narrow beam of continuous X-rays, electrons emerging from the slab were analyzed with a magnetic spectrometer. The magnet, G-M detector and related accessaries were turned round the aluminum slab in order to observe the electron spectra at different emerging angles, θ. The characteristics of X-rays used and the calibration of the spectrometer have been described in previous papers[5)7)].

3. EXPERIMENTAL RESULTS AND DISCUSSION

a) *Energy Spectrum of Emergent Electrons*

The energy spectra of electrons which emerged from the surface of aluminum slab were measured with the magnetic spectrometer for a definite solid angle at various emerging angles. The results are illustrated in Fig. 2 for 150 kV and 300 kV X-rays, where the ordinates correspond to the relative number of emerging electrons per cm^2 per steradian per keV interval for the same exposure of X-rays.

The total number N(E) of electrons which possess an energy E as they emerge from the surface of aluminum slab may be given by the integration of $N_\theta(E)$ over all emerging angles[7)]. After calculating N(E) for each electron energy of interest, the energy spectrum of emergent electrons was obtained. Fig. 3 shows the results given for 150,200, 250 and 300 kVp X-rays. In order to facilitate the comparison of electron spectra between bone and aluminum, the results obtained previously with bone are also repoltted in Fig. 3. It was found that there is a tolerable resemblance between bone and aluminum in the form of curves relating the energy spectrum of emergent electrons.

The mean energy of emergent electrons, which is defined by the weighted mean, $\int E \cdot N(E)\, dE / \int N(E)\, dE$ was estimated from the graphs given in Fig. 3. The results are tabulated in Table 1, showing that there is a fairly good agreement between bone and aluminum with respect to the mean energy of electrons.

Accordingly, it may be concluded that aluminum is fairly well equivalent to bone for X-rays in the

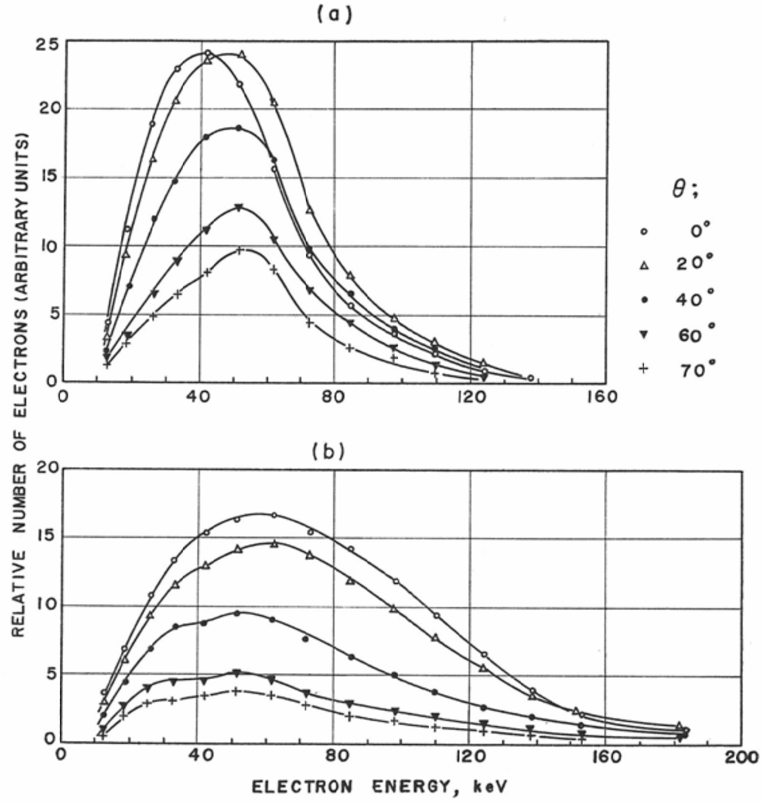

Fig. 2. Energy spectra of electrons emerging from the surface of aluminum slab at different emerging angles, for (a) 150 kVp X-rays and (b) 300 kVp X-rays.

Table 1 Mean Energy and Range of Electrons Ejected from Surface of Aluminum or Bone by X-ray Irradiation

X-rays (kVp)	Material	Electrons ejected from surface of material	
		Mean energy (keV)	Mean range in water (micron)
150	Aluminum	55	51
150	Bone	54	50
200	Aluminum	59	58
200	Bone	62	62
250	Aluminum	66	71
250	Bone	72	80
300	Aluminum	75	87
300	Bone	77	90

range from 150 kVp to 300 kVp so far as the energy of emitted electrons is concerned.

b) *Emergent Electron Flux*

Total number of electrons which emerge from the surface of irradiated slab per cm^2 per roentgen, that is, the emergent electron flux per roentgen may be estimated from the area under the distribution curve given in Fig. 3 for each quality of X-rays. The relative values derived from such a method are presented in Fig. 4, exhibiting considerable lack of agreement between aluminum and bone in res-

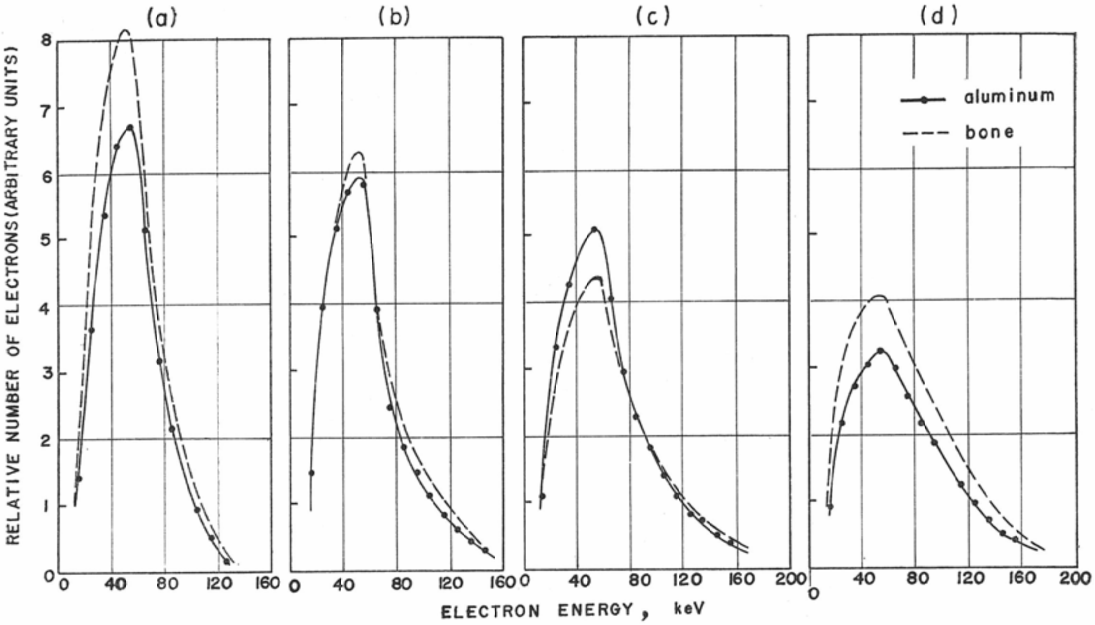

Fig. 3. Energy spectra of total electrons emerging from unit area of aluminum-or bone-slab for the same exposure of X-rays of 150 kVp(a), 200 kVp(b), 250 kVp(c)and 300 kVp(a).

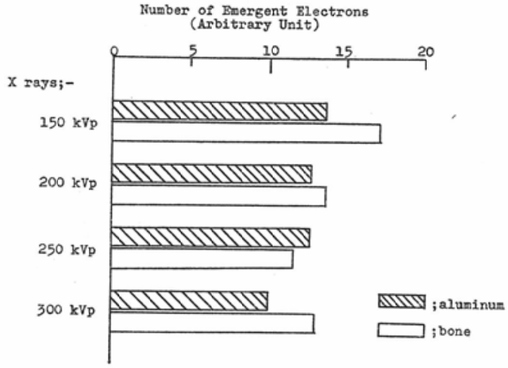

Fig. 4. Comparison of the magnitudes of emergent electron flux between aluminum and bone, for the same exposure of 150–300 kVp X-rays.

pect of the magnitude of the electron flux. With 200 kVp and 250 kVp X-rays the lack of agreement was as much as 10 percent, while it amounted to over 20 percent with 150 kVp or 300 kVp X-rays. Hence, it may be concluded that aluminum is not strictly equivalent to bone in respect of the numbers of emitted electrons for X-rays in the range from 150 kVp to 300 kVp.

The emergent electron flux varied with the quality of X-rays. This may be ascribed to the resultant variation of Compton electrons and photoelectrons involved in the flux; as X-ray energy is increased, the former increase on account of their range elongation while the latter decrease due to the rapid fall of photoelectric cross section. Detailed discussion will be made in section 4 relationg approximate calculations of emergent electron flux.

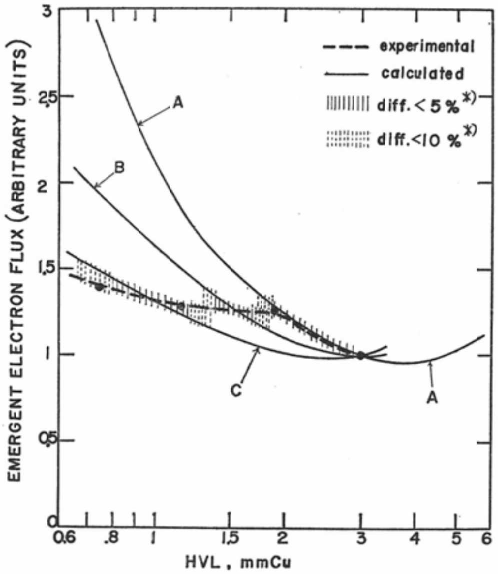

Fig. 5. Variation of the magnitude of emergent electron flux with X-ray qualites, for aluminum slab. The curves are normalized at HVL of 3mmCu. Approximate calculations using method I, II and III give curve A,B and C, respectively. *): Denote the region where the differences between calculated and experimental results are less than 5 % and 10%, respectively.

c) *Comparison of Experimental and Calculated Electron Flux*

Relative magnitudes of the electron flux derived from the experiment with aluminum, which are given in Fig. 4, are replotted in Fig. 5 as a function of the half-value layer (HVL). Three theoretical curves are also presented in Fig. 5, where curve A, B and C were derived from approximate calculations using method I, II and III, respectively (see section 4). It should be noted that the curves given in Fig. 5 are normalized at HVL of 3 mm.

A comparison of experimental and calculated curves leads to the following result; variation in the electron flux with X-ray quality can not be approximated solely by a single calculation method over all HVL range of interest. Curve A agrees with experimental one within 5% error in HVL range from about 1.9 mm to 3 mm Cu; curve B, in HVL range about 1.4—1.6 mm Cu; curve C, in HVL range about 0.8—1.2 mm Cu. Within 10% error, curve A agrees with experimental one in HVL range from about 1.8 mm to 3 mm Cu; curve B, in HVL range about 1.3—1.9 mm Cu; and curve C, in HVL range about 0.65—1.4 mm Cu.

Thus, approximation method I may be satisfactory for the estimation of the electron flux so far as the photoelectric contribution is not very great, but as this contribution is greatly increased, method II or III should be employed for the estimation of electron flux.

4. APPROXIMATE CALCULATION OF EMERGENT ELECTRON FLUX

Suppose an X-ray beam is incident normally on a plane slab of a material, and consider the number of electrons emerging from rear surface of the slab. If an elementary volume of the material emits isotropically N electrons per cubic centimeter per roentgen, the number n of electrons crossing an elementary area δA on the rear surface of the slab is approximated by the following expression:

$$n=\frac{1}{4}N \cdot R \cdot \delta A \qquad (1)$$

where the effective range of electrons is denoted by R, being taken to be 70% of the total rectified electron path-length[10)11)].

Now, consider a beam of photons of a given energy $h\nu$. Let ϕ denote the photon flux per roentgen; τ and σ, the linear absorption coefficients in the material for photoelectric and Compton effects; ρ, the density of the material in g/cm^3; R_1 and R_2, the effective electron ranges in g/cm^2 for photoelectrons and Compton electrons, respectively.

Then

$$n=\frac{\phi}{4}\left\{\frac{\tau \cdot R_1}{\rho} + \frac{\sigma \cdot R_2}{\rho}\right\}\delta A \qquad (2)$$

The photon flux per roentgen, ϕ, is given as below:

$$\phi=\frac{D_{air}}{(\mu_{en}/\rho)_{air}} \times (1/h\nu) \qquad (3)$$

where D_{air} denotes the absorbed dose in air in ergs/g per roentgen; $(\mu_{en}/\rho)_{air}$, the mass energy absorption coefficient in air in cm^2/g; and $h\nu$, the primary photon energy[6)].

In actual calculation, values for D_{air} and $(\mu_{en}/\rho)_{air}$ were taken from data given in ICRU Report 1962[4)], values for τ/ρ and σ/ρ in aluminum were referred to Grodstein's data[1)] and Nelms' data[8)], respectively. Values for R_1 and R_2 were estimated from Nelm's data[9)] by assuming that the initial energy of photoelectrons is nearly the same as the primary photon energy $h\nu$ and the initial mean energy of Compton electrons is given by $(\sigma_a/\sigma)h\nu$[4)]. The calculated values for n and ϕ are presented in Table 2.

More complicated calculation may be required for heterogeneous X-ray beam which consists of a continuous spectrum of photons. In the present paper, three approximation methods are proposed as below.

Method I: *Calulation Using Mean Energy of X-rays*

If a heterogeneous X-ray beam can be regarded as a beam of monoenergetic photons whose energy is the same as the mean energy of the former, the emergent electron flux may be determined directly by expression (2). Since the X-ray beams used in the present experiment possess the mean photon energies of about 70-, 90-, 110- and140-keV[7)], the relative magnitude of the electron flux per roentgen are given by the underlined figures in Table 2 for 150 kVp to 300 kVp X-rays. They are plotted in Fig. 5 as a function of HVL-values (curve A).

Method II: *Calulation Using Continuous Spectrum of X-rays*

If the continuous X-ray spectrum is taken into account instead of using simply the mean photon energy of the beam, the emergent electron flux may be determined by

$$n\sim\frac{1}{4}\Sigma\varphi(E)\ \Delta E\ \{(\tau/\rho)\ R_1+(\sigma/\rho)\ R_2\} \qquad (4)$$

where $\varphi(E)\Delta E$ represents the number of incident photons in each energy interval E to $E+\Delta E$. In practice, the photon energies were divided into 10 keV internal and calculations were carried out as shown in Table 3. Relative valus derived from this method are plotted in Fig. 5 as curve B.

Table 2 Calculation of Electron Flux Emerging from Surface of Aluminum, for Monoenergetic X-ray beam.

Photon energy (keV)	ϕ photons per cm^2 per roentgen, $\times 10^{-8}$	$(\tau/\rho)R_1$ $\times 10^6$	$(\sigma/\rho)R_2$ $\times 10^6$	$\phi\{(\tau/\rho)R_1+(\sigma/\rho)R_2\}$ electrons per cm^2 per roentgen, $\times 10^{-6}$
20	53.1	2230	0.34	11.8
30	124	1340	1.58	16.7
40	214	837	4.00	18.9
50	285	628	7.98	18.1
60	312	475	14.6	15.3
70	310	375	23.4	12.3 (2.89)
80	290	310	34.0	9.98
90	260	252	47.7	7.79 (1.82)
100	237	218	60.6	6.59
110	210	185	77.7	5.51 (1.29)
120	190	165	96.0	4.96
130	172	145	119	4.54
140	158	128	143	4.28 (1.00)
150	145	117	170	4.18
200	102	70.0	352	4.31
250	77.0	48.3	609	5.06
300	63.4	35.1	917	6.03

Table 3 Calculation of Electron Flux Emerging from Surface of Aluminum, for Unit Roentgen of 150 kV X-rays

Photon energy interval, (keV)	$\varphi(E)\Delta E$ photons per cm^2 per roentgen, $\times 10^{-8}$	$(\tau/p)R_1+(\sigma/p)R_2$ $\times 10^6$	$\varphi(E)\Delta E\{(\tau/\rho)R_1 + (\sigma/\rho) R_2\}$ electrons per cm^2 per roentgen, $\times 10^{-5}$
15—25	0.60	2230	1.34
25—35	9.10	1340	12.2
35—45	18.3	882	16.1
45—55	36.8	636	23.4
55—65	39.0	490	19.1
65—75	29.0	398	11.5
75—85	26.5	344	9.12
85—95	23.3	300	6.99
95—105	20.2	279	5.64
105—115	16.8	263	4.42
115—125	13.3	261	3.42
125—135	9.10	264	2.40
135—145	5.00	271	1.36

Total $=11.7\times 10^6$

Method III: *Calculation Using Angular Dependence of Electron Emission by Heterogeneous X-rays*

Above-mentioned two methods are based on the assumption that secondary electrons are emitted

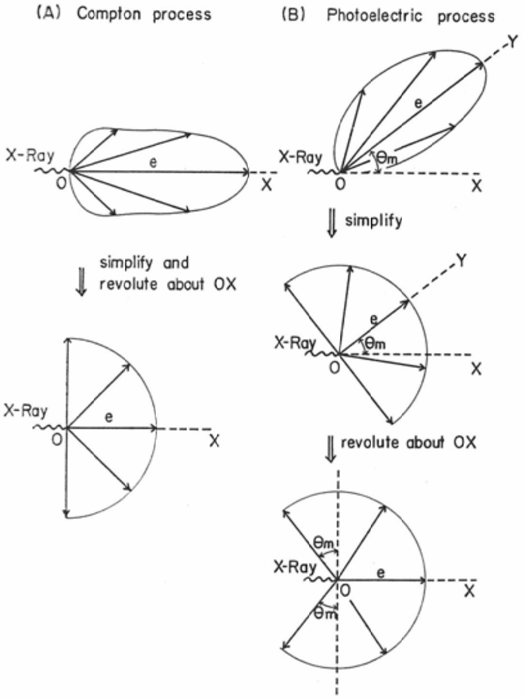

Fig. 6. Diagram illustrating the simplification of the angular dependence of secondary electron emission.

isotropically in the material by X-irradiation. The third calculation, however, will be made by taking account of the angular dependence of electron emission. Though the angular distribution of secondary electrons is given by a complicated formula[2)8)], the actual calculation is made by assuming simply that electrons are emitted uniformly only in the directions given in Fig. 6.

θ_m shown in Fig. 6 is the angle corresponding to the maximum intensity of emitted electrons, being given by expression (5), since the angular distribution of photoelectrons per unit solid angle is proportional to the term in bracket of expression (5).

$$\frac{d}{d\theta}\left[\frac{\sin^2\theta}{(1-\beta\cdot\cos\theta)^4}\right]=0 \tag{5}$$

where θ is the angle between the photon and the photoelectron, β is the ratio of the electron velocity to the speed of light[2)].

On the basis of the assumption given in Fig. 6, the solid angle, Ω, into which the photoelectrons are emitted can be determined by

$$\Omega=\int_{\theta=0^\circ}^{\pi/2+\theta_m} 2\pi\cdot\sin\theta\cdot d\theta=2\pi(1+\sin\theta_m). \tag{6}$$

Therefore, for a single photon incidence per cm^2, the number of emitted photoelectrons per steradian may be given by $\tau/2\pi(1+\sin\theta_m)$ instead of $\tau/4\pi$ used in method I or II. Similarly, the number of Compton electrons emitted into unit solid angle may be given by $\sigma/2\pi$ per photon per cm^2.

Thus, the emergent electron flux for the continuous X-ray spectrum may be given by

$$n \sim \frac{1}{2} \Sigma\varphi(E) \, \Delta E \left\{ \frac{1}{1+\sin\theta_m} (\tau/\rho) R_1 + (\sigma/\rho) R_2 \right\} \qquad (7)$$

Relative values were calculated by expression (7) for each quality of X-rays. The results are ploted in Fig. 5 as a function of HVL-values (curve C).

5. CONCLUSION

Experimental work was undertaken to know whether aluminum could be equivalent to bone with regard to the nature such as the secondary electron emission produced by heterogeneous X-rays. When an X-ray beam was incident normally on the front surface of a plane slab of aluminum, the energy spectra of electrons emerging from the rear surface of the slab were measured with a magnetic spectrometer at different emerging angles. The results were compared with those obtained previously with bone, and it was found that aluminum is fairly well equivalent to bone so far as the energy of emitted electrons is concerned, while it is not exactly equivalent to bone in respect of the numbers of emerging electrons.

Then, approximate calculations were made as to the emergent electron flux, being compared with experimental results. On the basis of the comparison between calculated and experimental results, it may be suggested that the angular dependence of electron emission should be considered in calculating the electron flux for high-Z material and for low-energy X-rays.

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